Exam

Spring 2021

Important: Please make sure that you answer all questions and that you properly explain your answers. For each step write the general formula (where relevant) and explain what you do. Not only the numerical answer. If you make a calculation mistake in one of the earlier sub-questions, you can only get points for the following subquestions if the formula and the explanations are correct!
60 points total

1. Short questions (10 points total)

   (a) In a bargaining game like the one we have seen in class the player with the higher discount factor is at an advantage. True or False. Explain in 2-3 sentences. (3 points)

   > **Solution:** True (1 point), having a higher discount factor (lower discount rate) means being more patient (1 point). If the discount factor is close to 1, the player is almost perfectly patient. The other player will then have to make a higher offer to quickly get to an agreement (1 point).

   (b) In a two-player matrix game, the process of iterated elimination of strictly dominated strategies will always lead to a pure-strategy Nash equilibrium. True or false? Explain in 2-3 sentences. Give an example. (4 points)

   > **Solution:** False (1 point), In many games (including games with multiple equilibria, games with mixed-strategy equilibria, and games with no strictly dominated strategies for either player), IESDS will not lead to PSNE (1 point for explanation). One specific example is matching Pennies - since no strategy for either player is strictly dominated, IESDS does not give any restriction on what strategies can be played (2 points for good example).

   (c) In each of the following examples, do bidders have private values, common values or both? Explain your answer. In each case, imagine the object is auctioned off to Wall Street investors. (3 points)

   - Shares of a stock in Mystery Enterprises Agricultural Technologies (MEAT), a fictional biotech start-up that sells "Mystery Meat" under the tagline "There is no mystery - it tastes like beef".
   - A lifetime supply of Mystery Meat
   - A hamburger

   > **Solution:** Common value means that the true value is the same for everyone, but people have different signals about the true value. Private value means that each bidder knows their true value, but this can be very different between people. In the case of the stocks, the private value is minimal (or equal to zero). The stocks can be resold to other investors. (1 point) In the case of the lifetime supply of mystery meat, bidders will have private values because they have different preferences for the meat supplement. However, because some of the meat could be resold to a

restaurant or other people, there is a common value aspect (1 point). In the case of a freshly cooked hamburger, it is purely private value, as some people might like hamburgers and are hungry and other people might not like them or are not currently hungry. (1 point).

2. Consider the following model of price competition. Firm 1 and 2 choose prices, $p_1 \geq 0$ and $p_2 \geq 0$ respectively. Total sales volume of firm $i = 1, 2$ is given by $q_i = 1 - p_i + p_j$, where j is the other firm's price. (If $1 - p_i + p_j < 0$, then the firm's sales are equal to zero.) Assume that the costs are equal to zero (which implies that the profit of firm i is equal to $p_i q_i$) and that both firms are profit maximizers. Firm 1 chooses p_1 without knowing p_2 but firm 2 has two options: (1) Choose p_2 without learning p_1; (2) Choose p_2 after learning p_1. Which option is better for firm 2? (Assume that the option chosen by firm 2 becomes common knowledge before the firms set their prices.) (10 points)

Solution: The best response for firm i is given by the following maximization problem:
$\max_{p_i \geq 0}(1 - p_i + p_j)p_i$ (2 points)
This yields: $p_i = \frac{1+p_j}{2} = BR_i(p_j) = \frac{1+p_j}{2}$ (1 points)
If firms choose prices independently, then the Nash equilibrium prices are given by the solution to the following system of equations: $p_1 = \frac{1+p_2}{2}$ and $p_2 = \frac{1+p_i}{2}$ (1 points)
This is solved at $p_1 = p_2 = 1$ (1 point)
Profit of firm2 at these prices is equal to 1. If firm 2 chooses its price after firm 1, and this is common knowledge, then firm 2 will choose:
$p_2 = \frac{1+p_i}{2}$ for any p_1. (2 points) Therefore, firm 1 will choose p_1 as the solution to the following problem:
$\max_{p_1 \geq 0}(1 - p_1 + \frac{1+p_1}{2})p_1$ (1 point)
The solution is $p_1 = 3/2$ and therefore $p_2 = 5/4$. (1 point)
Profit of firm 2 at these prices is equal to 25/16. Therefore firm 2 prefers to choose p_2 after learning p_1. (1 point)

3. Kim and Tim have to submit their homework assignment, but it is tedious to upload it to Absalon. Since they only need to upload one they hope that the other will do it. If both of them don't submit, they will fail the assignment. (13 points total)

Game A:

		Tim	
		S	W
Kim	S	$0, 0$	$-1, 1$
	W	$1, -1$	$-2, -2$

Game B:

		Tim	
		S	W
Kim	S	$0, 0$	$-1, 1$
	W	$1, -1$	$-10, -10$

(a) Find the pure and mixed Nash equilibria for game A. What are the probabilities that the players are mixing with? (3 points)

(b) Because they have failed a previous assignment, not submitting the next one would lead to them not being able to take the exam. This new situation is modeled in Game B. What is the mixed strategy equilibrium in Game B? Do they play "Wait" more or less often than in Game A? (2 points)

(c) What is the expected payoff for each player in the mixed strategy equilibrium in Game B? (2 points)

(d) Kim and Tim have several courses together, so they play Game B repeatedly. They don't want to fail so they decide to alternate between the two pure strategy equilibria. Assuming they have to submit an even amount of assignments what is the average payoff to both of them? Is this better or worse than what they can expect from playing the mixed-strategy equilibrium? Why? (Players to not discount over time.) (3 points)

(e) In the next semester Kim and Tim have forgotten which PSNE they played last time and neither of them realizes this until last minute. They both decide to throw a coin to decide whether to submit or not. What are the expected payoffs to Kim and Tim when they mix 50/50? How do these payoff compare with their expected payoffs when they play mixed strategies? Explain why these payoffs are the same or different from those in part c). (3 points)

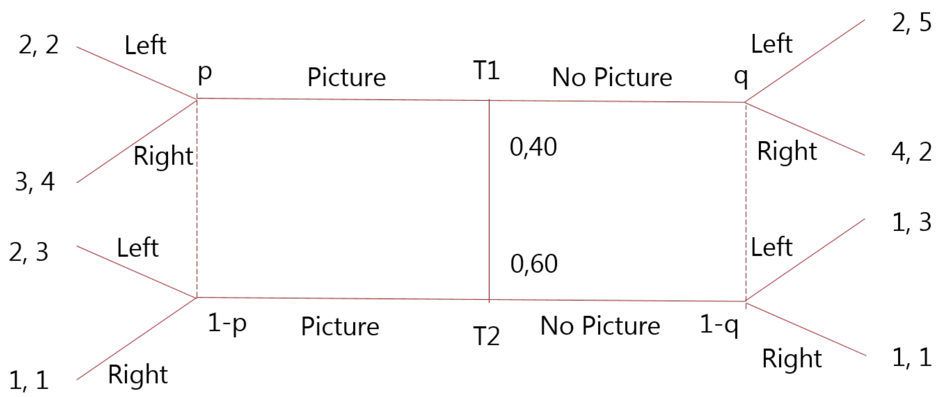
Solution: a) (Submit, Wait) and (Wait, Submit) are the pure strategy NE.
For game A they both mix with p=1/2 and q=1/2.
0p - 1(1 - p) = 1p - 10(1 - p) p = 9/10
0q - 1(1 - q) = 1q - 10(1 - q) q = 9/10
b) In the mixed-strategy Nash equilibrium Kim plays 9/10(Submit) + 1/10(Wait), and Tim plays 9/10 (Submit) + 1/10(Wait). Kim and Tim play Wait less often than in the previous game.
c) Kim's expected payoff = 9/10 - 10(1 - 9/10) = - 1/10.
Tim's expected payoff = 9/10 - 10(1 - 9/10) = - 1/10.
d) If Kim and Tim collude and play an even number of games where they alternate between (Submit, Wait) and (Wait, Submit), their expected payoffs would be 0. This is better than the mixed-strategy equilibrium, because their expected payoffs are - 1/10.
e) Kim's expected payoff = 1/2[(0 -1)/2] + 1/2[(1 - 10)/2] = - 5/2.
Tim's expected payoff = 1/2[(0 -1)/2] + 1/2[(1 - 10)/2] = - 5/2.
These expected payoffs are much worse than the mixed-strategy equilibrium. In this case both players are mixing with the wrong (that is, not the equilibrium) mixture. Neither player is best responding to the other's strategy, and in this situation, with the very real possibility of reaching the -10 payoff, the expected consequences are bad and they will probably fail. (Game B should be used because it refers to question c)).

4. Consider the following game between two sellers and a continuum of identical buyers. The sellers produce a homogenous good that the buyers value at v: The sellers contact the buyers by making simultaneously a price offer of p_i for each firm $i \in 1, 2$. The buyers observe the offers and buy from the seller with the lower price if that price does not exceed v: If the two prices are equal, then the market is equally split between the sellers. Find the pure strategy Nash equlibrium of this pricing game. Show how you get to the results by reasoning through your steps (proof). Tip: Think about the different types of prices each seller can charge and whether anyone has an incentive to deviate. (11 points)

Solution: Suppose $p_1^* < p_2^*$. If $p_1^* < v$, seller 1 can increase p_1 to $p_1^* + \epsilon$ to improve his payoff while still winning the sale. Hence, we consider $p_1^* \geq v$. In this case seller 1 makes a payoff of 0. However, by decreasing his price to $v - \epsilon$, seller 1 wins the sale and makes a payoff of $v - \epsilon > 0$.
Hence, $p_1^* < p_2^*$, (p_1^*,p_2^*) cannot be a pure strategy NE. Similarly, $p_2^* < p_1^*$ will not be a NE, by symmetry.
Hence, we just need to consider $p_1^* = p_2^*$.
If $p_1^* = p_2^* \in (v, \infty)$ then both sellers receive a payoff of 0: anyone of them can improve by setting $p_1 = v$ and gain a payoff of $v > 0$.
If $p_1^* = p_2^* \in (0, v]$ then any seller can improve by decreasing his price just a little bit to gain the entire market, which improves his payoff.
Hence, we are left with $p_1^* = p_2^* = 0$, which is the unique pure strategy NE. Given $p_j^* = 0$ seller i cannot improve his payoff since setting $p_i^* = 0$ gives a payoff of 0 whereas setting $p_i > 0$ results in losing his market and gaining the same payoff of 0.
In conclusion, the unique pure strategy NE is $p_1^* = p_2^* = 0$.

(2 points for the correct equilibrium, 4 points for naming the types of prices the sellers can choose (fx. $p_1^* < p_2^*$). 4 points for the explanation. 1 point for mentioning that the game is symmetric. This question can be solved with different reasoning, even if it isn't exaclty as written in the solution.)

5. Consider the following game G'. There are two types of senders on a popular dating app. T1 is a great cook, T2 only orders take-out. The receiver has to decide whether to swipe right on the sender after looking at their pictures. Some senders have pictures of themselves cooking in their fancy kitchen, some don't. The sender wants to score a date with the receiver. Possible messages are: P (Picture) and NP (No Picture). Possible actions for the receiver are: Swipe Left (L) and Swipe Right (R). (16 points total)



(a) Is G' a game of complete or incomplete information? Is it a dynamic or a static game? (2 points)

Solution: By definition, G is a game of incomplete information. (1 point) The receiver does not know which type the sender is. It is a dynamic game. (1 point)

(b) Find a separating equilibrium where T1 plays P and T2 plays NP. Show the steps of how to get to the solution. Explain your process. (5 points)

Solution: Check best response for receiver when T1 plays P and T2 plays NP. a*(P)=r, a*(NP)=l. Then check if any of the types want to deviate given the best response. No. PBE: (P NP, rl, p=1, q=0)

(c) Is there a pooling equilibrium on PP? Explain how you get to the solution. (5 points)

Solution: Pooling equilibrium: (PP, ll, p = 0.4, $q \in [0, 1]$).

(d) What if instead the sender writes "I love to cook". What do we call these types of games? (1 point)

Solution: This is a cheap talk game.

(e) What are the three conditions that need to be fulfilled for the message “I love to cook" to be effective and improve coordination? Explain what all three conditions mean in this dating example. (3 points)

Solution:

1. Sender types prefer different actions: The good cook wants to find a partner who also likes to cook. The bad cook wants to find someone who rather wants to eat take-out. (1 point)
2. Receiver types prefer different messages from different types: The receiver wants the good cook to say that they love cooking and the bad cook to say they like take out better. (1 point)
3. Sender and receiver incentives are somewhat aligned: Both types want to find someone who enjoys the same style of eating. (1 point)